Multi-fuzzy codes

Cite as: AIP Conference Proceedings **2336**, 040026 (2021); https://doi.org/10.1063/5.0045977
Published Online: 26 March 2021

V. Shijina

Multi-Fuzzy Codes

V. Shijina[a)]

Co-operative Arts and Science College, Madayi, Kannur, Kerala, India.

[a)]*Corresponding author: shijichan@gmail.com*

Abstract. Communication systems and storage devices of data are not completely reliable in application due to noise and further disturbances. In the twentieth century, coding theory came to light as a solution to all the troubles during the transmission of information. The study of coding theory had a specified beginning in the landmark paper by Shannon [1] and forged ahead to a very fruitful area of Mathematics. Afterwards, the ties between codes and fuzzy sets fabricated fuzzy coding theory and then, various favourable and advantageous studies are developed in this area. This paper intent to associate the concepts of multi-fuzzy sets to codes. A multi-fuzzy code (MF code) is defined as a multi-fuzzy subset of n-tuples over a set $\mathbb{F}$. Hamming distance between MF codes is introduced and its properties are presented.

INTRODUCTION

In modern times, there has been a growing exigency for digital data transmission and storage systems, that are reliable and consistently good in performance. This requirement has been increased in rate by the emergence of large scale high speed data networks for the exchange processing and storage of digital information in various spheres. Computer technology and communications were merged to design these systems.

In 1948, Claude Shannon [1] laid out the basic elements of communication in his paper "A Mathematical Theory of Communication". This paper marked the dawn of coding theory, which is a branch of study related to the transmission of data over the disturbing channels and the retrieval of distorted messages. Since its origination, coding theory has perceived remarkable progress. It has established many worldwide application in communication systems and storage technology. In 1981, von Kaenel [2] initiated the idea of fuzzy codes by extending the concepts of codes to fuzzy set theory. Fuzzy subset of $n-$tuples over a set $\mathbb{F}$ is called a fuzzy code. In 2000, Shum and De Gang [3] developed the notion of fuzzy linear code over finite fields. The fuzzy linear code is defined by using the theory of fuzzy linear space and the method of decode is also given. They also studied the fundamental properties of fuzzy cyclic codes. In 2020, S. A. Tsafack et al. [4] extended the concepts of linear codes and cyclic codes to fuzzy and explored some basic properties of these codes.

In 2010, S. Sebastian and T.V. Ramakrishnan [5] developed a new mathematical tool, called multi-fuzzy set, to represent multiplicity and uncertainty together. Multidimensional membership functions were utilized to develop these multi-fuzzy sets. They extended their study in many papers [6, 7, 8]. In 2013, this concept of multi-fuzzy sets was extended to multi-fuzzy rough sets by Anitha Sara Thomas and Sunil Jacob John [12]. Later, Yang et al. [9] developed the notion of multi-fuzzy soft set, by combining two mathematical tools multi-fuzzy sets and soft sets. This concept of multi-fuzzy soft set was later extended to many works [10, 11]. Recently, complex multi-fuzzy sets were introduced by Y Al-Qudah and N Hassan [13] and, the theory was developed through many papers [14, 15, 16].

This article endeavours to introduce multi-fuzzy codes by linking coding theory to multi-fuzzy sets. Hamming distance between multi-fuzzy codes is introduced and some results related to it are explored. This article will be arranged as follows: Section 2 gives a quick look at some basics of coding theory. It also describes the concepts related to multi-fuzzy sets. Section 3 proposes the multi-fuzzy codes. It also introduces hamming distance between multifuzzy codes and investigates its properties. In the last, the conclusion of the paper and future work related to this area are given.

PRELIMINARIES

Necessary backgrounds are provided in this section. Initially, the fundamental concepts of coding theory are given. Then, the idea of multi-fuzzy sets and standard operations on it are discussed.

International Conference on Computational Sciences-Modelling, Computing and Soft Computing (CSMCS 2020)
AIP Conf. Proc. 2336, 040026-1–040026-5; https://doi.org/10.1063/5.0045977
Published by AIP Publishing. 978-0-7354-4080-7/$30.00

Coding Theory

The definitions are from the textbook [17]. In general, informations are transmitting as a sequence of 0's and 1's. A 0 or a 1 is called a digit. A sequence of digits is known as a word. The length of the word is the number of digits in the word. A word is transmitted by sending its digits, one after the other, across a binary channel. Only two digits, 0 and 1, are used for transmission and hence the name binary channel. A binary code is a set C of words. A code having all its words of the same length is called a block code. This number is called the length of a code. The words that belong to a given code C will be called codewords. $|C|$ is the number of codewords in the code C.

Let $\mathbb{F} = \{0,1\}$ and let $\mathbb{F}^n$ be the set of all binary words of length n. Addition and multiplication of elements of $\mathbb{F}$ are defined as follows:

$$0+0=0, \quad 0+1=1, \quad 1+0=1, \quad 1+1=0$$

$$0\cdot 0=0, \quad 0\cdot 1=0, \quad 1\cdot 0=0, \quad 1\cdot 1=1$$

For the elements of $\mathbb{F}^n$, addition is defined componentwise, using the addition defined on $\mathbb{F}$ to add each component. Clearly, addition of two binary words of length n results in a binary word of length n. Using linear algebra terminology, we refer to an element of $\mathbb{F}$ as scalar. Then scalar multiplication of $\mathbb{F}^n$ is defined componentwise.

$$0\cdot w = \mathbf{0} \quad \textit{and} \quad 1\cdot w = w$$

where $\mathbf{0}$ is the zero word and w is any codeword. Clearly, $\mathbb{F}^n$ is closed under scalar multiplication. With these definitions of addition and scalar multiplication, $\mathbb{F}^n$ is a vector space.

Let v and w be words of length n. The **hamming distance** between v and w is the number of positions in which v and w disagree. We denote the distance between v and w by $d(v,w)$.

Multi-fuzzy Sets

Multi-fuzzy sets [5] Let U be a non empty set and let $\{L_i; i\in\mathbb{N}\}$ be a family of complete lattices where $\mathbb{N}$ is the set of all positive integers. A multi fuzzy set M in U is a set of ordered sequences

$$M = \{\langle u, m_1(u), m_2(u), ...\rangle; u\in U\}$$

where $m_i \in L_i^U$ for $i\in\mathbb{N}$. The function $m=(m_1,m_2,...)$ is called a multi membership function of multi fuzzy set M.

If the sequences of the membership function have only k terms, k is called dimension of M. Let $L_i=[0,1]$ for $i=1,2,...,k$, then the set of all multi fuzzy sets in X of dimension k is denoted by $M^kFS(U)$

Standard Operations on Multi-fuzzy Sets

Let $\{L_i : i\in\mathbb{N}\}$ be a family of complete lattices and

$$M = \{\langle u, m_1(u), ..., m_i(u), ...\rangle; u\in U,\ m_i\in L_i^U,\ i\in\mathbb{N}\}$$
$$N = \{\langle u, n_1(u), ..., n_i(u), ...\rangle; u\in U,\ n_i\in L_i^U,\ i\in\mathbb{N}\}$$

be multi-fuzzy sets in a nonempty set U. The following relations and operations are defined on multi fuzzy sets [6]:

1. **Subset**: M is a subset of N, denoted by $M\subseteq N$, iff $m_i(u)\le n_i(u)$ for every $u\in U$ and $i\in\mathbb{N}$.
2. **Equality**: M is equal to N, denoted by $M=N$, iff $m_i(u)=n_i(u)$ for every $u\in U$ and $i\in\mathbb{N}$.
3. **Union**: Union of M and N is defined by

$$M\cup N = \{\langle u, m_1(u)\vee n_1(u), , ..., m_i(u)\vee n_i(u), ...\rangle; u\in U,\ m_i,n_i\in L_i^U,\ i\in\mathbb{N}\}$$

4. **Intersection**: Intersection of M and N is defined by

$$M\cap N = \{\langle u, m_1(u)\wedge n_1(u), , ..., m_i(u)\wedge n_i(u), ...\rangle; u\in U,\ m_i,n_i\in L_i^U,\ i\in\mathbb{N}\}$$

MULTI-FUZZY CODES

Let $\mathbb{F} = GF(2)$ be a field of 2 elements and $\mathbb{F}^n$ be the n-dimensional vector space of n-tuples over $\mathbb{F}$.

Multi-fuzzy Word: Let $u = (u_1, u_2, ..., u_n)$ be any vector in $\mathbb{F}^n$. Then, a multi-fuzzy word (MF word) is a multi-fuzzy subset of $\mathbb{F}^n$, defined by

$$f_u = \{(w, f_u^1(w), f_u^2(w), ..., f_u^k(w),); w \in \mathbb{F}^n\}$$

where $f_u(w)$ is the multi-fuzzy membership functon defined by

$$f_u^j(w) = p_j^{n-d} q_j^d$$

for $w = (w_1, w_2, ..., w_n) \in \mathbb{F}^n$, $d = d(u, w)$, and p_j and q_j are real numbers with $p_j + q_j = 1$ for every $j = 1, 2, 3, ..., k$. For any $n \in \mathbb{N}$, the collection of all multi-fuzzy words is denoted by f^n.

Theorem: The function $\Phi : \mathbb{F}^n \to f^n$, defined by $\Phi(u) = f_u$ is a one-to-one correspondence.

Multi-fuzzy Code: If $C \subseteq \mathbb{F}^n$ is a code of length n, then $\Phi(C)$ is a subset of f^n and is called a multi-fuzzy code. If $a \in C$, f_a is a multi-fuzzy codeword (MF codeword) of $\Phi(C)$.

Hamming Distance Between MF Words: The hamming distance between two MF words $f_u, f_v \in f^n$ is defined as

$$HD(f_u, f_v) = \min_{j \in \mathbb{N}_k} \sum_{w \in \mathbb{F}^n} |f_u^j(w) - f_v^j(w)|$$

Hamming Distance of an MF Code: For any MF code $\Phi(C)$, the hamming distance is defined as

$$HD = \min\{HD(f_u, f_v); f_u, f_v \in \Phi(C), f_u \neq f_v\}$$

1. $f_u = f_v \implies HD(f_u, f_v) = 0$
   Proof: Suppose $f_u = f_v$, where
   $$f_u = \{(w, f_u^1(w), f_u^2(w), ..., f_u^k(w),); w \in \mathbb{F}^n\}$$
   and
   $$f_v = \{(w, f_v^1(w), f_v^2(w), ..., f_v^k(w),); w \in \mathbb{F}^n\}$$
   Then, $f_u^j(w) = f_v^j(w)$ for all $w \in \mathbb{F}^n$ and for every $j = 1, 2, 3, ..., k$. Therefore,
   $$HD(f_u, f_v) = \min_{j \in \mathbb{N}_k} \sum_{w \in \mathbb{F}^n} |f_u^j(w) - f_v^j(w)| = 0$$

2. $HD(f_u, f_v) = HD(f_v, f_u)$
   Proof: We have
   $$\begin{aligned} HD(f_u, f_v) &= \min_{j \in \mathbb{N}_k} \sum_{w \in \mathbb{F}^n} |f_u^j(w) - f_v^j(w)| \\ &= \min_{j \in \mathbb{N}_k} \sum_{w \in \mathbb{F}^n} |f_v^j(w) - f_u^j(w)| \\ &= HD(f_v, f_u) \end{aligned}$$

3. If $f_u \subseteq f_v \subseteq f_w$, then $HD(f_u, f_w) \geq HD(f_u, f_v)$ and $HD(f_u, f_w) \geq HD(f_v, f_w)$.
   Proof: Suppose $f_u \subseteq f_v \subseteq f_w$. Then, $f_u^j(x) \leq f_v^j(x) \leq f_w^j(x)$ for all $x \in \mathbb{F}^n$ and for every $j = 1, 2, 3, ..., k$. Then
   $$|f_u^j(x) - f_w^j(x)| \geq |f_u^j(x) - f_v^j(x)| \quad and \quad |f_u^j(x) - f_w^j(x)| \geq |f_v^j(x) - f_w^j(x)|$$
   Therefore,
   $$\begin{aligned} HD(f_u, f_w) &= \min_{j \in \mathbb{N}_k} \sum_{x \in \mathbb{F}^n} |f_u^j(x) - f_w^j(x)| \\ &\geq \min_{j \in \mathbb{N}_k} \sum_{x \in \mathbb{F}^n} |f_u^j(x) - f_v^j(x)| \\ &= HD(f_u, f_v) \end{aligned}$$

Similarly,

$$\begin{aligned} HD(f_u, f_w) &= \min_{j\in\mathbb{N}_k} \sum_{x\in\mathbb{F}^n} |f_u^j(x) - f_w^j(x)| \\ &\geq \min_{j\in\mathbb{N}_k} \sum_{x\in\mathbb{F}^n} |f_v^j(x) - f_w^j(x)| \\ &= HD(f_v, f_w) \end{aligned}$$

4. $HD(f_u \cap f_w, f_v \cap f_w) \leq HD(f_u, f_v)$
   Proof: We have

$$f_u \cap f_w = \{(x, f_u^1(x) \wedge f_w^1(x), f_u^2(x) \wedge f_w^2(x), ..., f_u^k(x) \wedge f_w^k(x)); x \in \mathbb{F}^n\}$$

and

$$f_v \cap f_w = \{(x, f_v^1(x) \wedge f_w^1(x), f_v^2(x) \wedge f_w^2(x), ..., f_v^k(x) \wedge f_w^k(x)); x \in \mathbb{F}^n\}$$

We have

$$|f_u^j(x) \wedge f_w^j(x) - f_v^j(x) \wedge f_w^j(x)| \leq |f_u^j(x) - f_v^j(x)|$$

for all $x \in \mathbb{F}^n$ and for every $j = 1, 2, 3, ..., k$. Therefore,

$$\begin{aligned} HD(f_u \cap f_w, f_v \cap f_w) &= \min_{j\in\mathbb{N}_k} \sum_{x\in\mathbb{F}^n} |f_u^j(x) \wedge f_w^j(x) - f_v^j(x) \wedge f_w^j(x)| \\ &\leq \min_{j\in\mathbb{N}_k} \sum_{x\in\mathbb{F}^n} |f_u^j(x) - f_v^j(x)| \\ &= HD(f_u, f_v) \end{aligned}$$

5. $HD(f_u \cup f_w, f_v \cup f_w) \leq HD(f_u, f_v)$
   Proof: We have

$$f_u \cup f_w = \{(x, f_u^1(x) \vee f_w^1(x), f_u^2(x) \vee f_w^2(x), ..., f_u^k(x) \vee f_w^k(x)); x \in \mathbb{F}^n\}$$

and

$$f_v \cap f_w = \{(x, f_v^1(x) \vee f_w^1(x), f_v^2(x) \vee f_w^2(x), ..., f_v^k(x) \vee f_w^k(x)); x \in \mathbb{F}^n\}$$

We have

$$|f_u^j(x) \vee f_w^j(x) - f_v^j(x) \vee f_w^j(x)| \leq |f_u^j(x) - f_v^j(x)|$$

for all $x \in \mathbb{F}^n$ and for every $j = 1, 2, ..., k$. Therefore,

$$\begin{aligned} HD(f_u \cup f_w, f_v \cup f_w) &= \min_{j\in\mathbb{N}_k} \sum_{x\in\mathbb{F}^n} |f_u^j(x) \vee f_w^j(x) - f_v^j(x) \vee f_w^j(x)| \\ &\leq \min_{j\in\mathbb{N}_k} \sum_{x\in\mathbb{F}^n} |f_u^j(x) - f_v^j(x)| \\ &= HD(f_u, f_v) \end{aligned}$$

CONCLUSION

In this article, a new idea of multi-fuzzy codes is introduced. It is defined by associating the multi-fuzzy set to the concept of codes. Moreover, hamming distance between multi-fuzzy codes is also introduced and some elementary properties are established.

FUTURE WORK

There is a large scope for future works concerning multi-fuzzy codes. The future work may introduce regular codes and may investigate the capacity of error correction of these codes in terms of its corresponding multi-fuzzy codes. The concepts of linear codes and cyclic codes may extend to multi-fuzzy sets to define multi-fuzzy linear codes and multi-fuzzy cyclic codes over some ring.

REFERENCES

1. C. E. Shannon, "A mathematical theory of communication," ACM SIGMOBILE mobile computing and communications review **5**, 3–55 (2001).
2. P. A. von KAENEL, "Fuzzy codes and distance properties," Fuzzy sets and systems **8**, 199–204 (1982).
3. K. Shum and C. De Gang, "Some note on the theory of fuzzy code," BUSEFAL (Bulletin Pour Les Sous Ensembles Flous et Leurs Applications) University of Savoie, France **81**, 132–136 (2000).
4. S. A. Tsafack, S. Ndjeya, L. Strüngmann, and C. Lele, "Fuzzy linear codes," Fuzzy Information and Engineering , 1–17 (2020).
5. S. Sebastian and T. V. Ramakrishnan, "Multi-fuzzy sets: An extension of fuzzy sets," Fuzzy Information and Engineering **3**, 35–43 (2011).
6. S. Sebastian and T. Ramakrishnan, "Multi-fuzzy extensions of functions," Advances in Adaptive Data Analysis **3**, 339–350 (2011).
7. S. Sebastian and T. Ramakrishnan, "Multi-fuzzy topology," Int. J. Appl. Math **24**, 117–129 (2011).
8. S. Sebastian and T. Ramakrishnan, "Multi-fuzzy subgroups," Int. J. Contemp. Math. Sciences **6**, 365–372 (2011).
9. Y. Yang, X. Tan, and C. Meng, "The multi-fuzzy soft set and its application in decision making," Applied Mathematical Modelling **37**, 4915–4923 (2013).
10. A. Dey and M. Pal, "Generalised multi-fuzzy soft set and its application in decision making," Pacific Science Review A: Natural Science and Engineering **17**, 23–28 (2015).
11. H.-d. Zhang and L. Shu, "Possibility multi-fuzzy soft set and its application in decision making," Journal of Intelligent & Fuzzy Systems **27**, 2115–2125 (2014).
12. A. S. Thomas and S. J. John, "Multi-fuzzy rough sets and relations," Annals of Fuzzy Mathematics and Informatics **7**, 807–815 (2014).
13. Y. Al-Qudah and N. Hassan, "Operations on complex multi-fuzzy sets," Journal of Intelligent & Fuzzy Systems **33**, 1527–1540 (2017).
14. Y. Al-Qudah and N. Hassan, "Complex multi-fuzzy relation for decision making using uncertain periodic data," International Journal of Engineering and Technology (UAE) **7**, 2437–2445 (2018).
15. Y. Al-Qudah and N. Hassan, "Complex multi-fuzzy soft expert set and its application," Int. J. Math. Comput. Sci **14**, 149–176 (2019).
16. Y. Al-Qudah and N. Hassan, "Bipolar fuzzy soft expert set and its application in decision making," International Journal of Applied Decision Sciences **10**, 175–191 (2017).
17. D. Hankerson, G. Hoffman, D. A. Leonard, C. C. Lindner, K. T. Phelps, C. A. Rodger, and J. R. Wall, *Coding theory and cryptography: the essentials* (CRC Press, 2000).